INTERCHANNEL MIXING OF HEAT CARRIERS IN A BUNDLE OF SPIRAL TUBES

B. V. Dzyubenko, P. A. Urbonas, and L. A. Ashmantas

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A study is made of the effect of different parameters on the effective diffusion coefficient in the intertube space of a heat exchanger. Flow is lengthwise over the oval-profiled tubes. A generalizing relation is obtained for calculating this coefficient.

Heat exchangers with spiral tubes are characterized by intensive interchannel mixing in the space between the tubes. This intensive mixing is the result of the spiral twisting of the heat carrier, leading to the appearance of transverse velocity components and to additional agitation and secondary circulation of the flow. The effective diffusion coefficients, determining the process of transverse mixing in such apparatus, is more than 10 times greater than the coefficient of turbulent diffusion on the axis of a circular pipe [1-5].

The mixing of a heat carrier in a bundle of spiral tubes has been studied by different methods. The works [1, 2] used the method of heat diffusion, based on a statistical Lagrangian description of the turbulence field, in the course of investigating the history of motion of individual particles continuously emitted by a point source. The empirical data on the dimensionless effective diffusion coefficient obtained in this case in the range of Reynolds numbers Re = $4.3 \cdot 10^3$–$8.2 \cdot 10^3$ is described well by the relation [2]:

$$k_{\mathrm{as}} = 0.0356\,(1 + 8.1\,\mathrm{Fr}_{\mathrm{m}}^{-0.278}), \tag{1}$$

where

$$\mathrm{Fr}_{\mathrm{m}} = s^2/dd_{\mathrm{e}}. \tag{2}$$

In determining k_{as}, use was made of the limiting solution of the Taylor equation for uniform and isotropic turbulence with long diffusion times.

The effective diffusion coefficients were determined in [1, 3-5] by the method of diffusion from linear heat sources on the basis of an Eulerian description of turbulent flow in a study of the motion of particles as a function of time and coordinates of points relative to which the medium was moving. In accordance with this method, one is examining free flow with slip of a homogenized medium containing distributed sources of volumetric energy release and drag at boundaries, while considering the displacement thickness of the boundary layer δ^*. The boundary layer is described by continuity equations, while the effect of homogenization is accounted for by introduction of the multiplier $(1-m)/m$. The following system of equations is used to solve the axisymmetric problem:

$$\rho u \frac{\partial u}{\partial x} = -\frac{dP}{dx} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\nu_{\mathrm{ef}}\frac{\partial u}{\partial r}\right) - \xi\frac{\rho u^2}{2d_{\mathrm{e}}}, \tag{3}$$

$$\rho u c_p \frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{\mathrm{ef}}\frac{\partial T}{\partial r}\right) + q_v\frac{1-m}{m}, \tag{4}$$

$$G = 2\pi m\int_0^{r_{\mathrm{k}}}\rho u r\,dr, \tag{5}$$

$$P = \rho RT \tag{6}$$

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with the boundary conditions

$$T(0, r) = T_{\text{in}},\ u(0, r) = u_{\text{in}},\ P(0, r) = P_{\text{in}}, \tag{7}$$

$$\left.\frac{\partial T(x, r)}{\partial r}\right|_{r=0} = 0,\ \left.\frac{\partial u(x, r)}{\partial r}\right|_{r=0} = 0, \tag{8}$$

$$\left.\frac{\partial T(x, r)}{\partial r}\right|_{r=r_k} = 0,\ \left.\frac{\partial u(x, r)}{\partial r}\right|_{r=r_k} = 0. \tag{9}$$

The coefficient D_t is linked with the effective coefficient of turbulent viscosity ν_{ef} and the effective coefficient of turbulent thermal conductivity λ_{ef} at $Pr_t = 1$ by the relations

$$D_t = \nu_{ef}/\rho, \tag{10}$$

$$D_t = \lambda_{ef}/\rho c_p, \tag{11}$$

determined from comparison of measured heat-carrier temperature fields with predicted fields obtained from numerical solution of system (3)-(6).

In [1, 3-5], the coefficients $\bar{k} = D_t/ud_e$, determined by the above method, were generalized by the relation

$$\bar{k}/k_{as} = 1 - \exp[-0.0504(2ax/d)], \tag{12}$$

where

$$a = 0.0745 + 11.37\text{Fr}_m^{-1} + 246\text{Fr}_m^{-2}. \tag{13}$$

Here it was assumed that the integral three-dimensional Lagrangian $\mathscr{L}_L$ and Eulerian $\mathscr{L}_E$ turbulence scales were equal and that the coefficient $\bar{k}$ depended only on the number Fr_m and the length of the bundle:

$$\bar{k} = k(\text{Fr}_m, x). \tag{14}$$

Although the use of Eqs. (1), (12), and (14) for generalizing empirical data on the coefficient $\bar{k}$ is valid given the assumptions made, it is necessary to establish how $\bar{k}$ is affected by such parameters as the length of the spiral tube bundle, the level of the initial turbulence, the Reynolds number, and the three-dimensional integral turbulence scales. In fact, according to [6], the ratio $\mathscr{L}_E/\mathscr{L}_L \neq 1$ on the axis of a circular pipe. This can also be seen in a bundle of spiral tubes. Evaluation of the length of the initial section on the basis of the works [7, 8] suggests that bundle length has a lesser effect on $\bar{k}$ than follows from Eq. (12). The range of Reynolds numbers covered in [1-5] was narrow and did not reveal the effect of the Re number on the coefficient k.

To solve these problems, we performed an experimental—theoretical study of the mixing of a heat carrier in bundles of spiral tubes 0.5 and 1.5 m long with numbers $Fr_m = 56$ and 220 in the range of $Re = 3.4 \cdot 10^3 - 3.8 \cdot 10^4$. The porosity range m = 0.49-0.511, while the level of initial turbulence $\varepsilon = 1-6\%$. The maximum dimension of the oval profile of the tubes was 12.2 mm. The study was conducted by the method of heat diffusion from linear sources. We solved an axisymmetric problem of equalization of temperature irregularities in a homogenized formulation. The experimental unit was described in [9]. The heat carrier was air. The effective diffusion coefficients were determined in a manner similar to the method in [1, 3-5]. Measurements of the temperature field in the outlet section of the bundle were compared with calculated fields. The limiting error of $\bar{k}$ was 25-50%. The temperature fields were calculated by solving nonlinear parabolic system (3)-(6) by the grid method using an explicit scheme. The test and calculated data were compared by a modification of the least-squares method and a statistical method [3] which allowed us to find the most reliable values of the dimensionless coefficient $\bar{k}$ and the confidence intervals of k from the test sample.

Figure 1 shows the results of the study. It follows from this data that the Reynolds number affects the coefficient k only in the region $Re < 10^4$, regardless of Fr_m. The level of turbulence at the inlet of the bundle has almost no effect on this coefficient. The certain increase in k for a bundle with $Fr_m = 220$ at $l = 0.5$ and $\varepsilon = 6\%$ (Fig. 1a) can be attributed to the effect of ε on k in the initial section. However, the fact that ε has a similar effect for a bundle with $Fr_m = 56$ and $l = 1.5$ m at $\varepsilon = 6.0\%$ but no effect for $Fr_m = 56$ and $l = 0.5$ m (Fig. 1b) allows us to attribute this phenomenon to the systematic error of the experiment. Since it is known that ε can affect the character of transfer only in the initial

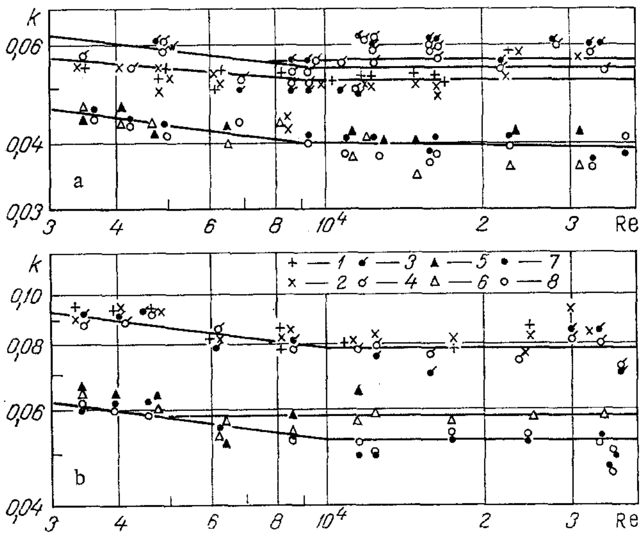


Fig. 1. Dependence of the effective diffusion coefficient on the Reynolds number, level of initial turbulence of the flow, and length of the bundle of spiral tubes with Fr_m = 220 (a) and Fr_m = 56 (b): 1, 2) test data for l = 0.5 m, ε = 1% and use of the least-squares method and statistical method, respectively; 3, 4) same, with l = 0.5 m and ε = 6%; 5, 6) same, with l = 1.5 m and ε = 1%; 7, 8) same, with l = 1.5 m and ε = 6%.

section of the flow — the length of which decreases with an increase in ε [6] — it becomes necessary to evaluate the length of the flow stabilization section by another method.

The data in [7, 8] can be used to evaluate the length of the initial section in the case of axisymmetric nonuniformity of the energy release over the bundle radius. Such an evaluation is necessary to establish the stabilized temperature profile for this case. The following relation [2, 7] determines the length of the section over which the profile of the velocity or temperature or the heat carrier is stabilized when the stream propagates on the scale of the maximum dimension of the tube oval in a bundle of spiral tubes:

$$(2ax/d)_b = 2{,}56. \tag{15}$$

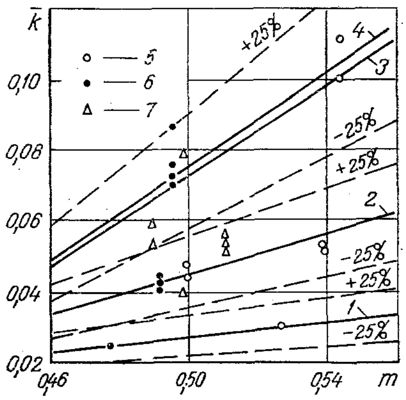
A length x_h/d_e [8] is required to develop a temperature nonuniformity in the core of the flow on the scale of the maximum dimension of the oval when the tubes are heated by an electrical current. This length is equal to the length of the initial thermal section in the case of uniform heating of the tube bundle over its radius. Then the length of the initial section, with axisymmetric nonuniformity of energy release, will be composed of these two lengths and will be determined by the relations

$$x_{h.b.}/d_e = 8.019 Fr_m^{0.226} \tag{16}$$

or

$$x_{h.b.}/d = 12{,}2a^{-1} Fr_m^{-0.275}. \tag{17}$$

Calculation of the length $x_{h.b.}$ with Eqs. (16), (17) shows that all of the bundles of spiral tubes examined in this article and in [1, 3-5] had lengths $l > x_{h.b.}$. This allows us to suggest that the coefficients k in these works were studied on sections of stabilized flow and that the resulting values of k are stabilized values k_s. The stratification of the empirical data seen here can be attributed to the accuracy of the determination of the coefficients k and to the effect of the porosity of the bundle. In fact, coefficients k for the same number Fr_m but different lengths were determined on bundles of different porosity due to the difficulty of ensuring the same assembly of tubes for bundles of different lengths. Thus, the porosity of the bundle to the heat carrier can be regarded as one of the determining parameters. In [10], it was also observed that the porosity of bundles of finnned rods affects the mixing rate. It turned out that the rate of mixing of the heat carrier in the



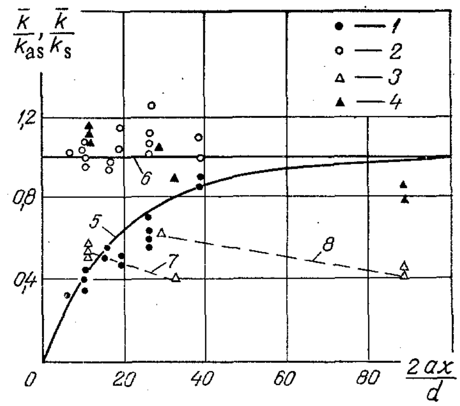


Fig. 2 Fig. 3

Fig. 2. Effect of porosity of a bundle of spiral tubes and the number Fr_m on the mean effective diffusion coefficient with the number $Re \geqslant 10^4$: 1-4) Eq. (19) at numbers Fr_m = 1050, 232, 64, and 56, respectively; 5, 6) test data [1, 3, 5]; 7) results of present experiment.

Fig. 3. Comparison of different methods of generalizing experimental data : 1, 2) test data from [1, 3, 5] analyzed in the form $\bar{k}/k_{as}$ and $\bar{k}/k_s$, respectively; 3, 4) the same, for our results; 5) Eq. (12); 6) Eq. (23); 7, 8) lines drawn through test points for bundles with Fr_m = 220 and 56, respectively.

bundle decreased with a decrease in the porosity of the bundle [10]. Thus, we may seek a criterional relationship for a stabilized value of the coefficient k_s in the form

$$k_s = k(\mathrm{Fr_m}, \mathrm{Re}, m). \tag{18}$$

Figure 2 shows results of tests by different authors in the form (18) for numbers $Re \geqslant 10^4$. In this Reynolds number region, the coefficient k_s is nearly independent of Re (see Fig. 1), and the test data (Fig. 2) is described well by the relation

$$k_s = 0.136\mathrm{Fr_m^{-0.256}} + 10\mathrm{Fr_m^{-0.66}}(m - 0.46). \tag{19}$$

In the range Re = $3.4 \cdot 10^3 - 10^4$, the Re number has an effect on the coefficient k_s similar to the effect of Re on the coefficient of turbulent diffusion in the circular pipe [6], and the following formula is valid:

$$k_s = 3.1623\,[0.136\mathrm{Fr_m^{-0.256}} + 10\mathrm{Fr_m^{-0.66}}(m - 0.46)]\,\mathrm{Re}^{-0.125}. \tag{20}$$

Equations (19), (20) were obtained in the parameter ranges: m = 0.46-0.56; Fr_m = 55-1080; ε = 1-6%; $l/d_e > x_{h.b.}/d_e$, where $x_{h.b.}$ is determined by Eqs. (16) and (17).

The coefficients k_s can be related to the coefficients k_{as}. Considering that these coefficients are proportional to the spatial scales of turbulence $\mathscr{L}_E$ and $\mathscr{L}_L$, respectively [6], we obtain

$$k_s/k_{as} = \mathscr{L}_E/\mathscr{L}_L. \tag{21}$$

Equation (21) can be used to determine the order of magnitude of the ratio $\mathscr{L}_E/\mathscr{L}_L$. Thus, for bundles with m = 0.475 and Re = $8 \cdot 10^3$, examined in [2], the ratio $\mathscr{L}_E/\mathscr{L}_L$ can be expressed as a function of the number Fr_m:

$$\mathscr{L}_E/\mathscr{L}_L = 0.785\mathrm{Fr_m^{-0.127}}. \tag{22}$$

Calculation with Eq. (22) shows that in the range Fr_m = 55-1080, the ratio $\mathscr{L}_E/\mathscr{L}_L$ changes within 0.5-0.3. This agrees with the results of Michelson's tests [6] for nearly uniform and isotropic turbulence in the core of a flow on the axis of a tube with a diameter of 0.2 m at Re = $2 \cdot 10^5 - 6 \cdot 10^5$. Michelson separately measured the longitudinal and Lagrangian correlation coefficients and determined that the mean value of the ratio $\mathscr{L}_E/\mathscr{L}_L$ = 0.6 in the above range of Reynolds numbers, which increases linearly with an increase in the flucutation velocity.

Figure 3 shows test data from different authors analyzed in the form of the functions $\bar{k}/k_{as} = f(2ax/d)$ and $\bar{k}/k_s = \varphi(2ax/d)$, where they are compared with Eqs. (12) and

$$\bar{k}/k_s = 1 \tag{23}$$

respectively. Equation (23) is valid for $2ax_{h.b.}/d \geqslant 24.4\ Fr_m^{-0.275}$. The spread of empirical data for Eq. (23) does not exceed the limiting error in the determination of $\bar{k}$ (Fig. 3). At the same time, the test results for bundles of spiral tubes 1.5 m long deviate markedly from Eq. (12).

The completed analysis and generalization of test data allowed us to establish a new criterional relation for determining effective diffusion coefficients. The relation can be used to calculate temperature fields with allowance for interchannel mixing of the heat carrier in bundles of spiral tubes.

NOTATION

Re, Reynolds number; Fr_m, criterion characterizing features of flow in a bundle of spiral tubes; Pr_t, turbulent Prandtl number; $k = D_t/ud_e$, dimensionless effective diffusion coefficient; D_t, effective diffusion coefficient; u, velocity; d_e, equivalent diameter; s, pitch of tube spiral; d, maximum dimension of tube profile; ρ, density; T, temperature; P, pressure; c_p, specific heat; G, mass flow rate of air; x, r, longitudinal and radial coordinates; q_v, density of energy release; m, porosity of bundle to heat carrier; r_k, radius of bundle; $\bar{k}$, mean value of coefficient k; k_s, stabilized value of coefficient k; l, bundle length; $\mathscr{L}_E$, $\mathscr{L}_L$, three-dimensional integral turbulence scales in Eulerian and Lagrangian descriptions of the flow; ε, level of turbulence of the flow at the bundle inlet; a, structure coefficient of the jet.

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